Calculus Q1 Test 2 Review Key

The Derivative gives us the slope of the tangent line

The Function gives us the point of tangency

1) $f(x) = x^2 + 8x + 16$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 8(x+h) + 16 - (x^2 + 8x + 16)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 8x + 8h + 16 - x^2 - 8x - 16}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + 8h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h(2x + h + 8)}{h}$$

$$f'(x) = \lim_{h \to 0} 2x + h + 8$$

$f'(x) = 2x + 8 \leftarrow$ Derivative

$f'(-2) = 2(-2) + 8$

$= -4 + 8$

$= 4 \leftarrow$ Slope when $x = -2$

Point

$f(-2) = (-2)^2 + 8(-2) + 16$

$= 4 - 16 + 16$

$= 4$

Point $= (-2, 4)$

Slope $= 4$

Equation

$(y - 4) = 4(x - (-2))$

OR

$\boxed{y - 4 = 4(x + 2)}$

2) $f(x) = \frac{2x}{x-3}$ when $x=1$

Point: $f(1) = \frac{2(1)}{1-3} = \frac{2}{-2} = -1$

$(1,-1)$

Slope $f'(x) = \lim_{h \to 0} \dfrac{\overset{2x+2h}{\frac{2(x+h)}{(x+h)-3}} - \frac{2x}{x-3}}{h} \cdot \dfrac{\frac{(x+h-3)(x-3)}{1}}{\frac{(x+h-3)(x-3)}{1}}$

$$= \lim_{h \to 0} \frac{(2x+2h)(x-3) - 2x(x+h-3)}{h(x+h-3)(x-3)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} - \cancel{6x} + \cancel{2xh} - 6h - \cancel{2x^2} - \cancel{2xh} + \cancel{6x}}{h(x+h-3)(x-3)}$$

$$= \lim_{h \to 0} \frac{-6\cancel{h}}{\cancel{h}(x+h-3)(x-3)} = \frac{-6}{(x+0-3)(x-3)}$$

$$f'(x) = \frac{-6}{(x-3)^2} \qquad f'(1) = \frac{-6}{(1-3)^2} = \frac{-6}{4} = \frac{-3}{2}$$

Point $= (1,-1)$

$\boxed{\text{Equation: } y+1 = -\frac{3}{2}(x-1)}$

Slope $= -\frac{3}{2}$

3) $f(x) = \sqrt{3x-3}$ when $x=4$

Point: $f(4) = \sqrt{3(4)-3} = \sqrt{9} = 3$

$(4,3)$

Slope: $f'(x) = \lim_{h \to 0} \frac{\sqrt{3(x+h)-3} - \sqrt{3x-3}}{h} \cdot \frac{\sqrt{3x+3h-3} + \sqrt{3x-3}}{\sqrt{3x+3h-3} + \sqrt{3x-3}}$

$$= \lim_{h \to 0} \frac{\cancel{3x} + 3h \cancel{-3} - (\cancel{3x} \cancel{-3})}{h\left(\sqrt{3x+3h-3} + \sqrt{3x-3}\right)}$$

$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3x+3h-3} + \sqrt{3x-3}\right)}$$

$$= \frac{3}{\sqrt{3x-3} + \sqrt{3x-3}}$$

$$f'(x) = \frac{3}{2\sqrt{3x-3}}$$

$$f'(4) = \frac{3}{2\sqrt{3(4)-3}} = \frac{3}{2\sqrt{9}} = \frac{3}{2 \cdot 3} = \frac{3}{6} = \frac{1}{2}$$

Point: $(4,3)$

$\boxed{\text{Equation: } y-3 = \frac{1}{2}(x-4)}$

Slope: $\frac{1}{2}$

4) $f(x) = 3x^2 - 4x + 2$ when $x = 2$

Point: $f(2) = 3(2)^2 - 4(2) + 2$

$(2,6)$ $f(2) = 12 - 8 + 2$

$f(2) = 6$

Slope: $f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 4(x+h) + 2 - (3x^2 - 4x + 2)}{h}$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-4x} - 4h \cancel{+2} \cancel{-3x^2} \cancel{+4x} \cancel{-2}}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 - 4h}{h}$$

$$= \lim_{h \to 0} 6x + 3h - 4 = 6x - 4$$

Derivative

$f'(x) = 6x - 4$

$f'(2) = 6(2) - 4$

$f'(2) = 8$

Point: $(2, 6)$

Slope: 8

$\boxed{\text{Equation: } y - 6 = 8(x - 2)}$

5) $f(x) = \dfrac{-x}{2x+3}$ when $x = 3$

Point: $f(3) = \dfrac{-3}{2(3)+3} = \dfrac{-3}{9} = -\dfrac{1}{3}$

$\left(3, -\frac{1}{3}\right)$

Slope:
$$f'(x) = \lim_{h\to 0} \frac{\dfrac{-(x+h)}{2(x+h)+3} - \dfrac{-x}{2x+3}}{h} \cdot \frac{\dfrac{(2x+2h+3)(2x+3)}{1}}{\dfrac{(2x+2h+3)(2x+3)}{1}}$$

$$= \lim_{h\to 0} \frac{-(x+h)(2x+3) + x(2x+2h+3)}{h(2x+2h+3)(2x+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{-2x^2} \cancel{-3x} \cancel{-2xh} - 3h + \cancel{2x^2} + \cancel{2xh} + \cancel{3x}}{h(2x+2h+3)(2x+3)}$$

Derivative

$$= \lim_{h\to 0} \frac{-3\cancel{h}}{\cancel{h}(2x+2h+3)(2x+3)} = \frac{-3}{(2x+3)^2} \qquad f'(x) = \frac{-3}{(2x+3)^2}$$

$$f'(3) = \frac{-3}{(2(3)+3)^2} = \frac{-3}{81} = \frac{-1}{27}$$

Point: $\left(3, -\frac{1}{3}\right)$

Slope: $-\dfrac{1}{27}$

$$\boxed{\text{Equation: } y + \frac{1}{3} = \frac{-1}{27}(x-3)}$$

6) $f(x) = \sqrt{x^2+1}$ $\qquad x=0$

Point:

$(0,1)$

$f(0) = \sqrt{0^2+1}$

$f(0) = \sqrt{1}$

$f(0) = 1$

Slope:

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h} \cdot \frac{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}$$

$$= \lim_{h\to 0} \frac{(x+h)^2+1 - (x^2+1)}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} \cancel{-x^2} \cancel{-1}}{h\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{2x\cancel{h} + h^{\cancel{2}}}{\cancel{h}\left(\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right)} = \lim_{h\to 0} \frac{2x+h}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}$$

$$f'(x) = \frac{2x}{2\sqrt{x^2+1}} \qquad f'(0) = \frac{0}{2} = 0$$

Point: $(0,1)$

Slope $= 0$

Equation: $\boxed{y-1 = 0(x-0)}$

$y-1=0$

$\boxed{y=1}$